UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN
FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA
TIPO DE EXAMEN Y/O EVALUACIÓN:
EXTRAORDINARIO (*Extra Exam*)

MATERIA/UNIDAD DE APRENDIZAJE: *Temas Selectos de Optimización*

LEARNING UNIT: Selected Topics on Optimización (in English)
SEMESTER: August – December 2025 (Fall)
ACADEMY: Statistics and Operations Research (*Estadística e Investigación de Operaciones*).
INSTRUCTOR: Dr. Roger Z. Ríos Mercado (ID 090969)

DIRECTIONS.- Answer the following questions and/or exercises in the answer sheet. Do not write in this sheet

SECTION 1: QUESTIONS (50 POINTS)

Answer and justify your answer.

1. [*UT1: Combinatorial optimization*; 5 pts] Define a combinatorial optimization problem.
2. [*UT1: Combinatorial optimization*; 5 pts] Define and explain what a brute-force enumeration method is for solving a combinatorial optimization problem.
3. [*UT2: Heuristics*; 5 pts] Define and explain what a heuristic method is for solving combinatorial optimization problems.
4. [*UT2: Constructive heuristics;*5 pts] What is a constructive heuristic?
5. [*UT2: Local search heuristics;* 5 pts] Explain clearly what a local search heuristic is for a combinatorial optimization problem.
6. [*UT2: Constructive heuristics;*5 pts] Describe in detail the nearest neighbor heuristic for solving the Traveling Salesman Problem. Illustrate your idea with an example or drawing.
7. [*UT2: Local search heuristics for the TSP;* 5 pts] Describe in detail the nearest insertion heuristic for solving the Traveling Salesman Problem. Illustrate your idea with an example or drawing.
8. [*UT1: Combinatorial optimization*; 5 pts] Is it true that the Traveling Salesman Problem is hard to solve? Explain.
9. [*UT2: Constructive heuristics;*5 pts] Do constructive heuristics guarantee to find a feasible solution to a given combinatorial optimization problem? Justify your answer.
10. [*UT2: Local search heuristics;* 5 pts] Explain the difference between the “first found” and “best found” strategies employed in local search heuristics.

SECTION 2: PROBLEMS (50 POINTS)

11. The p-Dispersion Problem (pDP) is defined as follows. Given a collection of points in the plane, $V = \{1, 2, ..., n\}$, where the distance d_{ij} between every pair of points i, j in V is known, and given a known positive integer number p, the goal is to decide a subset of p points of V, in a such a way that these points are as disperse (far away from each other) as possible. In other words, find a subset X of cardinality p such that a dispersion objective function is maximized. For a feasible solution given by $X = \{ v_1, v_2, ..., v_p \}$, the dispersion function is computed as: $f(X) = \min_{i,j \in X} \{d_{ij}\}$, that is, the minimum distance among all pairs of points in subset X. The problem consists of finding the subset X that maximizes function $f(X)$, that is, that maximizes the minimum distance in set X. In Figure 1 below, there is an instance with 13 points and correspondent distance matrix D.

    The following questions refer to the pDP instance described in Figure 1, assuming $p = 3$.

    (a) [*UT1: Combinatorial optimization;* 5 pts] Is $X^{(1)} = \{3, 6, 9, 11\}$ a feasible solution? Justify your answer.

    (b) [*UT1: Combinatorial optimization;* 5 pts] Is $X^{(2)} = \{1, 5, 12\}$ a feasible solution? Justify your answer.

    (c) [*UT1: Combinatorial optimization;* 8 pts] Among the following three solutions, sort them from best to worst. Justify your answer.
    $X^{(3)} = \{3, 6, 8\}$,
    $X^{(4)} = \{6, 7\}$,
    $X^{(5)} = \{2, 5, 11\}$.

    (d) [*UT2: Constructive heuristics;* 10 pts] Starting from scratch, design a constructive heuristic for finding a feasible solution to the pDP with n points and p dispersion points. Show very clearly and precision each step of your heuristic either in pseudocode or flow chart, explaining carefully what your heuristic does.

    (e) [*UT2: Constructive heuristics;* 6 pts] Illustrate how your heuristic works by applying it step by step in the example (Figure 1) to build a feasible solution to the problem. Give details of the computation in each iteration. Was this solution better than solution $X^{(3)}$ from (c)?

    (f) [*UT2: Local search heuristics;* 10 pts] Given a feasible solution to the CP, design a local search for the problem. It is sufficient to describe **very clearly** how you define your move/neighborhood.

    (g) [*UT2: Local search heuristics;* 6 pts] Illustrate how your local search works starting from the following feasible solution $X^{(3)} = \{3, 6, 8\}$. Do at least one **complete** iteration under the "best move" strategy. Did the solution improve?

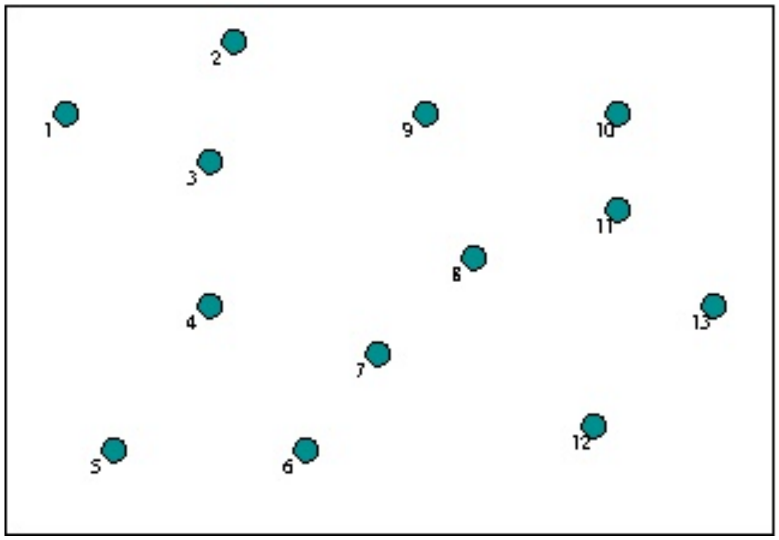

Figure 1: Original set of points V in the plane. Each point is labeled by a number. Distance matrix D is given below.

$$D = \left(\begin{array}{c|ccccccccccccc}
- & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\
\hline
1 & 0 & 15 & 13 & 20 & 28 & 34 & 33 & 36 & 30 & 46 & 47 & 51 & 56 \\
2 & & 0 & 10 & 22 & 35 & 35 & 29 & 27 & 17 & 33 & 35 & 44 & 46 \\
3 & & & 0 & 12 & 25 & 25 & 21 & 23 & 18 & 34 & 34 & 39 & 45 \\
4 & & & & 0 & 16 & 14 & 15 & 22 & 24 & 38 & 35 & 34 & 42 \\
5 & & & & & 0 & 16 & 23 & 34 & 38 & 50 & 47 & 40 & 51 \\
6 & & & & & & 0 & 10 & 21 & 30 & 38 & 33 & 24 & 36 \\
7 & & & & & & & 0 & 11 & 20 & 28 & 23 & 20 & 28 \\
8 & & & & & & & & 0 & 13 & 17 & 13 & 17 & 20 \\
9 & & & & & & & & & 0 & 16 & 18 & 30 & 29 \\
10 & & & & & & & & & & 0 & 8 & 26 & 18 \\
11 & & & & & & & & & & & 0 & 18 & 11 \\
12 & & & & & & & & & & & & 0 & 14 \\
13 & & & & & & & & & & & & & 0
\end{array}\right)$$

Figure 2: Distance matrix D of example. Since D is symmetric, only the upper triangular part is shown. Just as an example, if p = 3, one possible solution could be subset X = {1, 2, 3}. For this subset, $f(X) = \min\{ d_{12}, d_{13}, d_{23}\} = \min\{15, 13, 10\} = 10$.